

Adiabatic dynamics in open quantum critical many-body systems

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The purpose of this work is to understand the effect of an external environment on the adiabatic dynamics of a quantum critical system. By means of scaling arguments we derive a general expression for the density of excitations produced in the quench as a function of its velocity and of the temperature of the bath. We corroborate the scaling analysis by explicitly solving the case of a one-dimensional quantum Ising model coupled to an Ohmic bath.

The understanding of the non-equilibrium dynamics of strongly correlated quantum systems is one of the most challenging problems of modern condensed matter physics. Interest on this subject has been revived recently by unprecedented experimental breakthroughs in the context of cold atomic gases (see e.g [1]). Non-equilibrium conditions in cold gases can be realized controllably in various ways, e.g. by a proper choice of the initial state, or by changing the Hamiltonian in time. In view of the variety of situations that can be studied in this context, it is of paramount importance to find paradigmatic situations that allow the general features of the non-equilibrium dynamics of many-body systems to be understood. One such paradigm is obtained when the parameters of a quantum system close to a quantum phase transition are varied in time in such a way as to traverse the quantum critical point. Because of the vanishing of the gap Δ at criticality, a finite density of defects is generated, no matter how slow is the quench, as first shown in Ref. [2, 3]. The density of defects is a *universal* scaling function of the quench velocity v , as in the Kibble-Zurek (KZ) mechanism [4] originally derived for classical continuous phase transition. In addition to its intrinsic interest, this problem is relevant to adiabatic quantum computation [5] and quantum annealing [6].

The intense theoretical activity following Refs. [2, 3] has clarified several important issues (see [7] and references therein) on the adiabatic, phase coherent dynamics of closed many-body critical systems. Closed systems, however, are only idealizations: any quantum system is weakly coupled to an environment inducing relaxation and dephasing. This observation motivated a series of recent studies, in particular on the effect of classical [9] and quantum [10] noise acting uniformly on a quantum Ising chain, and on the effect of local noise on a disordered Ising chain with up to 20 spins [11]. However, the most natural and important question remained answered: *to what extent is it possible to describe universally the production of defects in an adiabatic quench in the presence of dissipation and dephasing?* In this Letter, we answer

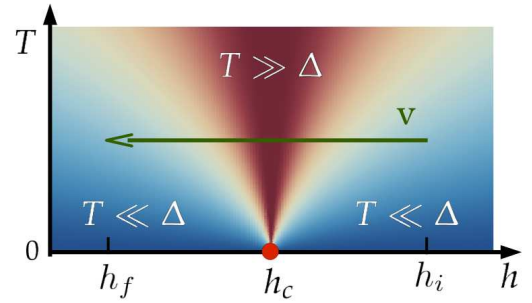


Figure 1: A sketch of the crossover diagram at finite temperatures due to the presence of the quantum critical point, where the gap $\Delta \rightarrow 0$. The system traverses the quantum critical region in a time t_{QC} .

this question by showing with a general scaling analysis that *even in the presence of an external environment* the adiabatic dynamics of open critical systems is governed by *universal scaling laws* (with modified exponents). To support this statement, we solve by means of quantum kinetic equations the adiabatic dynamics of a quantum Ising chain coupled to a local external environment, leading to the relaxation of all quasiparticle modes.

Scaling analysis of defect formation. At equilibrium, all the characteristic features of a quantum phase transition leave an imprint at low temperatures, leading, close to the quantum critical point, to a crossover at temperatures $T \sim \Delta$. For $T \ll \Delta$, the low frequency dynamics can be described semiclassically in terms of quasi-particle excitations, while in the *quantum critical region* (red region in Fig.1) $T \gg \Delta$ and quasi-particle excitations no longer exist [8]. The purpose of the analysis below is to discuss the universal fingerprints left on the nonequilibrium dynamics by the quantum phase transition in the presence of a bath. For this sake, we now start by presenting a scaling analysis describing the influence of a bath at temperature T on the production of defects.

We start by considering a linear quench of the control parameter h from an initial value h_i to a final one h_f

across the critical point h_c at a speed v . The system is initially in equilibrium with the bath, whose temperature T is kept constant during the quench. This is sketched in Fig.1. The passage through the quantum critical region ($T \gg \Delta$) leads to substantial heating effects because of the relatively high bath temperature with respect to the characteristic energy scale Δ for the system excitations. We shall see that, as long as this is the most important time interval for the bath-systems interaction, only the low energy details of the system spectrum matter, and universality holds.

The universality of the defect production in the presence of a bath follows from two conditions, discussed below. First of all, the density of excitations \mathcal{E} can be written as

$$\mathcal{E} \simeq \mathcal{E}_{KZ} + \mathcal{E}_{inc} \quad (1)$$

where \mathcal{E}_{KZ} is the coherent contribution, present also in a closed system, and \mathcal{E}_{inc} is the incoherent one due to the presence of the bath. For a closed system, the density of excitations was found to scale as $\mathcal{E}_{KZ} \propto v^{d\nu/(z\nu+1)}$ [2, 3], where d is the spatial dimension of the system and ν , z are the correlation length and the dynamical critical exponents respectively. Moreover we assume that the bath does not influence the system in the semiclassical regions ($T \ll \Delta$). Hence, we consider thermal excitations predominantly created inside the quantum critical region, where $T \gg \Delta$, at a rate $\tau^{-1} \propto \alpha T^\theta$, α being the system-bath coupling constant. The dynamics for the population of the (excited) mode k is thus governed by the rate equation $\frac{d}{dt}P_k = -\tau^{-1}(P_k - P_k^{th}(h_c))$, where $P_k^{th}(h_c)$ is the critical thermal equilibrium distribution inside the quantum critical region. Integrating this rate equation we obtain $P_k(h_f) \sim (1 - e^{-\tau^{-1}t_{QC}})P_k^{th}(h_c)$, where $t_{QC} = 2T^{1/\nu}v^{-1}$ is the time spent going through the quantum critical region (see Fig.1), whose boundaries are given by $T \sim \Delta \sim |h - h_c|^{\nu z}$. Integrating over all k -modes we obtain:

$$\mathcal{E}_{inc} \propto \left(1 - e^{-\frac{t_{QC}}{\tau}}\right) \int dE E^{\frac{d}{z}-1} P_k^{th}(h_c), \quad (2)$$

where we used the scaling of the critical energy $E \propto k^z$. Finally, since the thermal distribution P_k^{th} is a function of E/T , changing variable to E/T leads to

$$\mathcal{E}_{inc} \propto \alpha v^{-1} T^{\theta + \frac{d\nu+1}{\nu z}}, \quad (3)$$

valid in the limit $T^{1/\nu z} \ll v\tau$. The v^{-1} scaling of \mathcal{E}_{inc} is directly related to the time spent inside the quantum critical region. The crossover from the coherent to the incoherent defect production is reached when $\mathcal{E}_{inc} \simeq \mathcal{E}_{KZ}$, giving

$$v_{cross} \propto \alpha^{\frac{\nu z+1}{\nu(z+d)+1}} T^{(1+\frac{(\theta-1)\nu z}{\nu(z+d)+1})(1+\frac{1}{\nu z})}. \quad (4)$$

The different scaling with v of \mathcal{E}_{KZ} and \mathcal{E}_{inc} implies that for fast quenches, $v > v_{cross}$, the KZ contribution domi-

nates, while for slower quenches, $v < v_{cross}$, the incoherent contribution due to the thermalization induced by the bath is the most important.

Eqs. (1),(3),(4) represent the generalization of the scaling laws given in Refs.[2, 3] to the case of open quantum critical systems. They are in principle amenable of an experimental verification and constitute the key result of this work. In the case of a quantum Ising chain, which we discuss in detail in the second part of the Letter, the previous expressions specialize as follows. The time spent within the quantum critical region scales as $t_{QC} = 2Tv^{-1}$, since $\nu = z = 1$. We consider an Ohmic bath that acts as a random external magnetic field on each site (see Eq. (5)). We find that the relaxation time in the quantum critical region scales as $\tau^{-1} \simeq \alpha T^2$ (see Fig. 2), *i.e.*, $\theta = 2$, in agreement with a Fermi golden rule argument. It follows that the contribution to the defect production induced by the bath and the crossover velocity scale as $\mathcal{E}_{inc} \propto \alpha v^{-1} T^4$ and $v_{cross} \propto \alpha^{2/3} T^{8/3}$, respectively (see Fig. 3).

Quantum Ising Model and kinetic equations. To support Eqs. (3) and (4) we now study the physics of a quantum quench for a quantum Ising model coupled locally to a set of Ohmic baths. The locality of the system-bath coupling causes the breaking of the translational symmetry of the closed system, hence allowing the discussion of the quench dynamics in the presence of relaxation of all elementary excitations. Notice that no qualitative features are expected to emerge in the case where the baths are correlated over a finite distance because (in the scaling limit) the correlation length is the largest lengthscale in the problem, and details of bath correlations over microscopic distances should not matter. It is also important to observe that long time correlations induced by the bath may change the universality class of the transition [15]. Despite the intrinsic interest of this issue, which found recently application in the context of the physics of cold atoms [12], we will not consider it here. Therefore, we will further assume that the bosons have a non-zero inverse lifetime $\gamma \ll T$ which provides a natural cutoff-time for the bath correlation functions.

The model we consider is then defined by the Hamiltonian

$$H = -\frac{J}{2} \sum_j^N \{ \sigma_j^x \sigma_{j+1}^x + [h(t) + X_j] \sigma_j^z \} + H_B. \quad (5)$$

It consists of a chain of N spins (σ^x and σ^z are Pauli matrices) with an Ising interaction and subject to a transverse magnetic field $h(t)$. The bath couples to σ^z , with $X_j = \sum_\beta \lambda_\beta (b_{\beta,j}^\dagger + b_{\beta,j})$, where $b_{\beta,j}^\dagger (b_{\beta,j})$ are the creation (annihilation) operators for the bosonic bath modes coupled to the j -th spin. The bath Hamiltonian is $H_B = \sum_{j,\beta} \omega_\beta b_{\beta,j}^\dagger b_{\beta,j}$. The system-bath coupling is chosen to have Ohmic spectral densities $\sum_\beta \lambda_\beta^2 \delta(\omega - \omega_\beta) = 2\alpha\omega \exp(-\omega/\omega_c)$, where ω_c is a high-energy cut-

off [13, 14]. In the case of no coupling to the bath ($\alpha = 0$) the system has a QPT at $h_c = 1$, and for $h < h_c$ a spontaneous magnetization along x appears. The gap $\Delta = |h - h_c|$ induces at finite temperature a V-shaped crossover phase diagram [8], as sketched in Fig. 1.

We now analyze the problem by deriving a quantum kinetic equation which allows us to calculate the density of defects produced after the quench. This procedure allows us to describe the effect of the environment also in regimes which are beyond the applicability of the scaling laws deduced above, where universality is not expected to hold. In order to describe the dynamics of (5) it is first convenient to map the spins onto spinless fermions by means of a Jordan-Wigner transformation. In momentum space, the Hamiltonian (5) reads:

$$H = \sum_{k>0} \Psi_k^\dagger \hat{\mathcal{H}}_k \Psi_k + \frac{1}{\sqrt{N}} \sum_{k,q} \Psi_k^\dagger \hat{\tau}^z \Psi_{k+q} X_q + H_B, \quad (6)$$

where $\hat{\mathcal{H}}_k = -[\cos(k) + h(t)]\hat{\tau}_z + \sin(k)\hat{\tau}_y$, $\hat{\tau}$ are Pauli matrices in the Nambu space defined by the two-component fermion $\Psi_k^\dagger = (c_k^\dagger \ c_{-k})$. When $X_q = 0$, the subspaces of the different k modes are decoupled, the Hamiltonian is quadratic and can be diagonalized by a Bogoliubov transformation yielding $H = \sum_{k>0} \Lambda_k (\eta_k^\dagger \eta_k - \eta_{-k} \eta_{-k}^\dagger)$ with $\Lambda_k = \sqrt{1 + 2h \cos k + h^2}$. The interaction with the bath causes a mixing of modes with different momenta. We determine the density of excitations by deriving a quantum kinetic equation for the Green's function using the Keldysh technique [16]. This kinetic equation will be expressed in terms of the fermion lesser Green's function $\hat{G}^<$, which is a 2×2 matrix in Nambu space with components $-i[G_k^<(t, t)]_{i,j} \equiv \langle \Psi_{k,j}^\dagger(t) \Psi_{k,i}(t) \rangle$. Using a self-consistent Born approximation for the bath-mediated scattering of k -modes, valid for weak bath-system coupling $\alpha \ll 1$, and the Markov approximation (justified by the assumption of a cutoff time for the bosonic modes), we get:

$$\partial_t \hat{G}_k^< + i[\hat{\mathcal{H}}_k, \hat{G}_k^<] = \frac{1}{N} \sum_q \hat{\tau}^z (\hat{1} + i\hat{G}_q^<) \hat{D}_{qk} \hat{G}_k^< + \hat{\tau}^z \hat{G}_q^< \hat{D}_{kq}^\dagger (\hat{1} + i\hat{G}_k^<) + H.c. \quad (7)$$

where we neglect irrelevant Lamb shifts. Here $\hat{D}_{qk} = i \int_0^\infty ds g^>(s) \hat{\mathcal{U}}_q^\dagger(t, t-s) \hat{\tau}^z \hat{\mathcal{U}}_k(t, t-s)$, where $g^>(t) = -i \langle X_q(t) X_q(0) \rangle$, and $\hat{\mathcal{U}}_k(t_0, t)$ is the evolution operator satisfying $i\partial_t \hat{\mathcal{U}}_k = \hat{\mathcal{H}}_k(t) \hat{\mathcal{U}}_k$. Parameterizing $-i\hat{G}_k^< = 1/2[\hat{1} - (1 - 2P_k)\hat{\tau}_z + C_k\hat{\tau}^+ + C_k^*\hat{\tau}^-]$ after the Bogoliubov transformation which diagonalizes $\hat{\mathcal{H}}_k$, one finds that $P_k = \langle \eta_k^\dagger \eta_k \rangle$ and $C_k = \langle \eta_{-k} \eta_k \rangle$. Therefore, the density of defects produced can be expressed as

$$\mathcal{E} = \frac{-i}{2N} \sum_{k>0} \text{Tr} \left[(\hat{1} + \hat{\tau}^z) \hat{G}_k^< \right] = \frac{1}{N} \sum_{k>0} P_k. \quad (8)$$

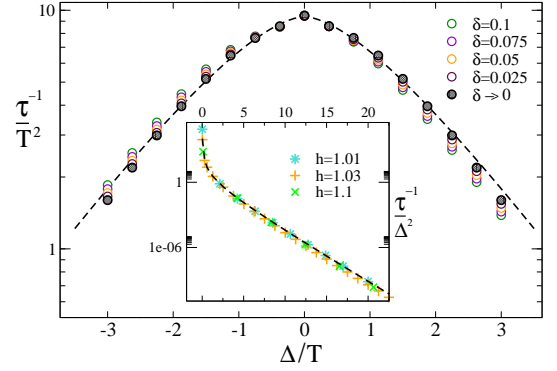


Figure 2: τ^{-1}/T^2 as a function of Δ/T (negative values correspond to $h < h_c$). Data are relative to different distances $\delta = \sqrt{T^2 + (h - h_c)^2}$ from the QPT. Close to the critical point ($\delta \rightarrow 0$) all curves collapse into a unique scaling function. Dashed line is a fit $a(1 + b\Delta/T) \exp\{-\Delta/T\}$ with $a \simeq 9.4(6)$ and $b \simeq 0.9(3)$. Inset shows the data collapse of τ^{-1}/Δ^2 as a function of Δ/T for different values of h : $\tau^{-1} \propto \exp\{-\Delta/T\}$ for $T \ll \Delta$.

By solving the kinetic equation (7) we are able to calculate \mathcal{E} in Eq. (8), and thus to analyze the Kibble-Zurek mechanism in an Ising chain coupled to a bath. The results of this analysis are presented below.

Kinetic equation and the scaling regime. As discussed in the first part of this Letter, in order to determine the scaling at finite temperature we need to know the relaxation time τ for the excitations. To this end, it is sufficient to consider only the dynamics of the populations P_k in the kinetic equation and neglect the off-diagonal components. By linearizing the kinetic equation around the equilibrium (Fermi) distribution, one gets $\partial_t \delta P = -\mathcal{R}(h, T) \delta P$, where $\delta P = (P_{k_1} - P_{k_1}^{th}, \dots, P_{k_{N/2}} - P_{k_{N/2}}^{th})$. The characteristic relaxation times of the system are the inverse eigenvalues of \mathcal{R} . The leading asymptotics at long times for all populations P_k is governed by the inverse of the smallest eigenvalue λ_1 of \mathcal{R} , $\tau \equiv \lambda_1^{-1}$. By numerical inspection, we find that $\tau^{-1} \sim \alpha T^2 f(\frac{\Delta}{T}) e^{-\Delta/T}$ (see Fig 2).

The scaling obtained for the relaxation time, together with an explicit integration of Eq. (2) for the Ising model, lead to

$$\mathcal{E}_{inc} \simeq \frac{\log 2}{2\pi} T \left(1 - e^{-2T/(\tau v)} \right), \quad (9)$$

which, in the limit $2T/\tau v \ll 1$, confirms the scaling result given in Eq. (3). In Fig. 3 the solution of the kinetic equation is compared with the ansatz given in Eqs. (1) and (9). The agreement is excellent, confirming our scaling approach. The crossover value of v which signals the transition from the coherent- to the incoherent-dominated defect production obeys the power-law scaling given by Eq. (4). At lower quench rates the full expression, Eq. (9), is needed for an accurate comparison with

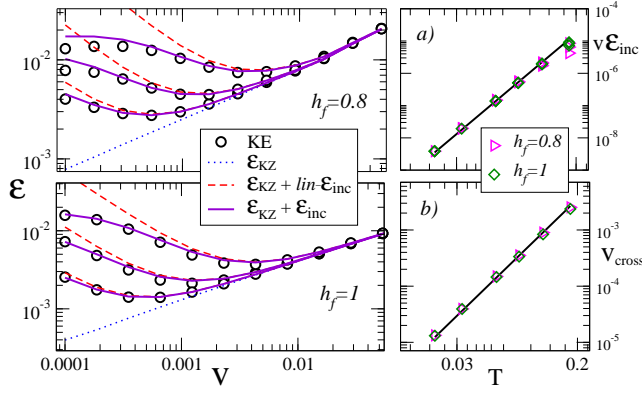


Figure 3: Left: density of excitations \mathcal{E} versus the quench velocity v . Circles are obtained by integrating the kinetic equation (KE); solid and dashed lines are Eq. (1) with \mathcal{E}_{inc} given by (9) and its linearized expression, respectively. KZ scaling $\mathcal{E}_{KZ} \propto \sqrt{v}$ [2] is plotted for comparison as dotted lines. Plots refer to $N = 400$ and $\alpha = 0.01$ and $T = 0.15, 0.1, 0.07$ (from top to bottom). The final values of magnetic field are $h_f = 0.8$ (outside the quantum critical region for the values of T considered) and the critical point $h_f = 1$. For $h_f = 1$ we considered $t_{QC} = v^{-1}T$ since only half of the quantum critical region is crossed. a) Data collapse of \mathcal{E}_{inc} obtained from the kinetic equations by considering only incoherent thermal transitions (rhs of (7)); data refer to both $h_f = 0.8, 1$ and $10^{-4} \lesssim v \lesssim 10^{-3}$. The fit verifies the scaling (3) $\mathcal{E}_{inc} \propto v^{-1}T^4$. In b) the scaling of v_{cross} is obtained equating \mathcal{E}_{inc} from kinetic equations to \mathcal{E}_{KZ} for $h_f = 0.8, 1$; the fit confirms the scaling predicted by (4), $v_{cross} \propto T^{8/3}$.

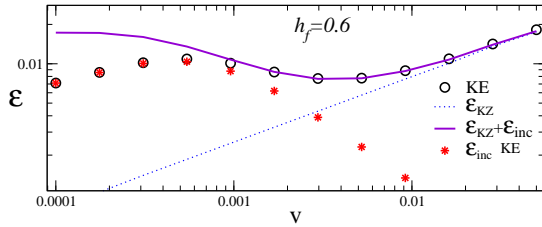


Figure 4: Same as left panel of Fig. 3, but for $h_f = 0.6$; curves are relative to $T = 0.15$ only. Stars represent the density of excitations produced incoherently by the bath and are obtained by integrating the kinetic equations and ignoring the unitary evolution term $i[\hat{\mathcal{H}}_k, \hat{G}_k^<]$, responsible for the coherent excitation process. The non-monotonous behavior is due to the non-critical relaxation mechanism, that is more relevant at low v . In the regime of slow v scaling no longer holds and only thermal excitations contribute to \mathcal{E} .

the solution of the kinetic equation.

On lowering the final value h_f of the field, the agreement with the scaling ansatz becomes worse at low quench rates v (see Fig. 4). This is due to a non-critical relaxation mechanism that depends strongly on the details of the energy spectrum for $h_f < h < h_c$: Once the system leaves the quantum critical region, entering the semiclassical region where $T \ll \Delta$, the bath starts

to relax-out the excitations previously created; if time spent in the semiclassical region is long enough all excitations disappear. Outside of the weak coupling regime, when the relaxation time and quench time are comparable, scaling is expected to be recovered in thermodynamic quantities, such as total energy, heat, entropy and work done on the system [17].

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